

Sample Question Paper - 4
Mathematics (041)
Class- XII, Session: 2021-22
TERM II

Time Allowed: 2 hours

Maximum Marks: 40

General Instructions:

1. This question paper contains three sections – A, B and C. Each part is compulsory.
2. Section - A has 6 short answer type (SA1) questions of 2 marks each.
3. Section – B has 4 short answer type (SA2) questions of 3 marks each.
4. Section - C has 4 long answer-type questions (LA) of 4 marks each.
5. There is an internal choice in some of the questions.
6. Q 14 is a case-based problem having 2 sub-parts of 2 marks each.

Section A

1. Evaluate: $\int \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) dx$ [2]

OR

Evaluate: $\int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx$

2. Solve the initial value problem: $(xe^{y/x} + y) dx = x dy$, $y(1) = 1$ [2]
3. For what value of λ are the vectors \vec{a} and \vec{b} perpendicular to each other? Where $\vec{a} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{b} = 3\hat{i} + 2\hat{j} - \lambda\hat{k}$ [2]
4. Find the direction cosines of the line $\frac{x-2}{2} = \frac{2y-5}{-3}$, $z = -1$. Also, find the vector equation of the line. [2]
5. An experiment succeeds twice as often as it fails. Find the probability that in the next six trials, there will be at least 4 successes. [2]
6. A bag contains 4 red and 5 black balls, a second bag contains 3 red and 7 black balls. One ball is drawn at random from each bag, find the probability that the balls are of the same colour. [2]

Section B

7. Evaluate $\int \frac{2x+1}{\sqrt{3x+2}} dx$ [3]
8. Solve the following differential equation. [3]
 $\cos^2 x \frac{dy}{dx} + y = \tan x$

OR

Form the differential equation of the family of circles touching the y - axis at the origin.

9. For any two vectors \vec{a} and \vec{b} prove that: $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$. [3]
10. A line makes angles α, β, γ and δ with the diagonals of a cube, prove that $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta = \frac{4}{3}$ [3]

OR



Find the shortest distance between the lines whose vector equations are

$$\vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \text{ and } \vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}).$$

Section C

11. Evaluate: $\int \frac{dx}{\sin x(3+2\cos x)}$. [4]

12. Draw a rough sketch of the region $f(x, y) : y^2 \leq 5x, 5x^2 + 5y^2 \leq 36\}$ and find the area enclosed by the region using method of integration. [4]

OR

Using integration, find the area of the region enclosed between the two circles $x^2 + y^2 = 4$ and $(x - 2)^2 + y^2 = 4$.

13. Find the shortest distance between the given lines. $\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$, [4]
 $\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$

CASE-BASED/DATA-BASED

14. In an office three employees Govind, Priyanka and Tahseen process incoming copies of a certain form. Govind process 50% of the forms, Priyanka processes 20% and Tahseen the remaining 30% of the forms. Govind has an error rate of 0.06, Priyanka has an error rate of 0.04 and Tahseen has an error rate of 0.03. [4]



Based on the above information, answer the following questions.

- The manager of the company wants to do a quality check. During inspection he selects a form at random from the days output of processed forms. If the form selected at random has an error, the probability that the form is NOT processed by Govind is
- Let A be the event of committing an error in processing the form and let E_1 , E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form. The value of

$$\sum_{i=1}^3 P(E_i | A)?$$

Solution

MATHEMATICS BASIC 041

Class 12 - Mathematics

Section A

1. Let $I = \int \tan^{-1} \left(\frac{3x-x^3}{1-3x^2} \right) dx$

Also let $x = \tan \theta$ then $dx = \sec^2 \theta d\theta$

$$I = \int \tan^{-1} \left(\frac{3 \tan \theta - \tan^3 \theta}{1 - 3 \tan^2 \theta} \right) \sec^2 \theta d\theta$$

$$= \int \tan^{-1} (\tan 3\theta) \sec^2 \theta d\theta$$

$$= \int 3\theta \sec^2 \theta d\theta$$

$$= 3 \left[\theta \int \sec^2 \theta d\theta - \int (1 \int \sec^2 \theta d\theta) d\theta \right]$$

$$= 3 \left[\theta \tan \theta - \int \tan \theta d\theta \right]$$

$$= 3 \left[\theta \tan \theta + \log \sec \theta \right] + C$$

$$= 3 \left[\theta \tan^{-1} x - \log \sqrt{1+x^2} \right] + C$$

$$I = 3x \left[\tan^{-1} x - \frac{3}{2} \log |1+x^2| \right] + C$$

OR

Let $I = \int \frac{\sec^2 \sqrt{x}}{\sqrt{x}} dx \dots (i)$

Also let $\sqrt{x} = t$ then, we have

$$d(\sqrt{x}) = dt$$

$$\Rightarrow \frac{1}{2\sqrt{x}} dx = dt$$

$$\Rightarrow dx = 2\sqrt{x} dt$$

$$\Rightarrow dx = 2t dt \quad [\because \sqrt{x} = t]$$

Putting $\sqrt{x} = t$ and $dx = 2t dt$ in equation (i) we get

$$I = \int \frac{\sec^2 t}{t} \times 2t dt$$

$$= 2 \int \sec^2 t dt$$

$$= 2 \tan t + c$$

$$= 2 \tan \sqrt{x} + c$$

$$\therefore I = 2 \tan \sqrt{x} + c$$

2. We have, $(xe^{y/x} + y) dx = x dy$

$$\Rightarrow \frac{dy}{dx} = e^{y/x} + \frac{y}{x}$$

This is a homogeneous differential equation. Putting $y = vx$ and $\frac{dy}{dx} = v + x \frac{dv}{dx}$ it reduces to

$$v + x \frac{dv}{dx} = e^v + v$$

$$\Rightarrow x \frac{dv}{dx} = e^v$$

$$\Rightarrow e^{-v} dv = \frac{dx}{x}, \text{ if } x \neq 0$$

Integrating both side with respect to x

$$\Rightarrow \int e^{-v} dv = \int \frac{1}{x} dx$$

$$\Rightarrow -e^{-v} = \log |x| + C$$

$$\Rightarrow -e^{-y/x} = \log |x| + C \dots (i)$$

It is given that $y(1) = 1$ i.e. when $x = 1, y = 1$. Putting $x = 1, y = 1$ in (i), we get: $-e^{-1} = C$

Putting $C = -\frac{1}{e}$ in (i), we get

$$-e^{-y/x} = \log |x| - \frac{1}{e}$$

$$\Rightarrow e^{-y/x} = \frac{1}{e} - \log |x| \Rightarrow -\frac{y}{x} = \log (1 - e \log |x|) = -1 \Rightarrow y = x - x \log (1 - e \log |x|)$$

Hence, $y = x - x \log (1 - e \log |x|)$, is the solution of the given equation.



3. Since, \vec{a} and \vec{b} are perpendicular

$$\therefore \vec{a} \cdot \vec{b} = 0$$

$$\Rightarrow (2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} + 2\hat{j} - \lambda\hat{k}) = 0$$

$$\Rightarrow (2)(3) + (3)(2) + (4)(-\lambda) = 0$$

$$\Rightarrow 6 + 6 - 4\lambda = 0$$

$$\Rightarrow 12 - 4\lambda = 0$$

$$\Rightarrow -4\lambda = -12$$

$$\Rightarrow \lambda = \frac{-12}{-4}$$

$$\Rightarrow \lambda = 3$$

4. The Cartesian equations of the given line are

$$\frac{x-2}{2} = \frac{2y-5}{-3}, z = -1$$

These above equations can be re-written as

$$\frac{x-2}{2} = \frac{2y-5}{-3} = \frac{z+1}{0} \text{ or, } \frac{x-2}{2} = \frac{y-5/2}{-3/2} = \frac{z+1}{0}$$

This shows that the given line passes through the point $(2, \frac{5}{2}, -1)$ and has direction ratios proportional to $2, -\frac{3}{2}, 0$. So, its direction cosines are

$$\frac{2}{\sqrt{2^2 + (-\frac{3}{2})^2 + 0^2}}, \frac{-3/2}{\sqrt{2^2 + (-\frac{3}{2})^2 + 0^2}}, \frac{0}{\sqrt{2^2 + (-\frac{3}{2})^2 + 0^2}} \text{ or, } \frac{2}{5/2}, \frac{-3/2}{5/2}, 0$$

$$\text{or, } \frac{4}{5}, -\frac{3}{5}, 0$$

The given line passes through the point having a position vector $\vec{a} = 2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}$ and is parallel to the vector $\vec{b} = 2\hat{i} - \frac{3}{2}\hat{j} + 0\hat{k}$.

Therefore, its vector equation is

$$\vec{r} = \left(2\hat{i} + \frac{5}{2}\hat{j} - \hat{k}\right) + \lambda \left(2\hat{i} - \frac{3}{2}\hat{j} + 0\hat{k}\right)$$

5. $p = 2x, q = x$

$$p + q = 1 \Rightarrow 2x + x = 1$$

$$\Rightarrow 3x = 1 \Rightarrow x = \frac{1}{3}$$

$$\therefore p = \frac{2}{3}, q = \frac{1}{3}$$

$$P(\text{at least 4 successes}) = P(X = 4) + P(X = 5) + P(X = 6)$$

$$= C(6, 4) p^4 q^2 + C(6, 5) p^5 q + p^6$$

$$= C(6, 4) \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + C(6, 5) \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^6$$

$$= 15 \left(\frac{2}{3}\right)^4 \left(\frac{1}{3}\right)^2 + 6 \left(\frac{2}{3}\right)^5 \left(\frac{1}{3}\right) + \left(\frac{2}{3}\right)^6$$

$$= \left(\frac{2}{3}\right)^4 \left(5 + 2 \times \frac{2}{3} + \frac{4}{9}\right)$$

$$= \left(\frac{2}{3}\right)^4 \left(\frac{15+12+4}{9}\right)$$

$$= \left(\frac{2}{3}\right)^4 \left(\frac{31}{9}\right) = \frac{496}{729}$$

6. Given:

Bag A = (4R + 5B) balls

Bag B = (3R + 7B) balls

Therefore, required probability is given by,

$$P(\text{balls of same colour}) = P(\text{both red}) + P(\text{both black})$$

$$= \frac{4}{9} \times \frac{3}{10} + \frac{7}{10} \times \frac{5}{9}$$

$$= \frac{12}{90} + \frac{35}{90}$$

$$= \frac{47}{90}$$

Section B

$$7. \text{ Let } I = \int \frac{2x+1}{\sqrt{3x+2}} dx$$

$$\text{Let } 2x + 1 = \lambda(3x + 2) + \mu$$

On equating the coefficients of like powers of x on both sides, we get

$$3\lambda = 2 \text{ and } 2\lambda + \mu = 1$$

$$\begin{aligned}
&\Rightarrow \lambda = \frac{2}{3} \text{ and } 2 \times \frac{2}{3} + \mu = 1 \\
&\Rightarrow \lambda = \frac{2}{3} \text{ and } \mu = \frac{-1}{3} \\
\therefore I &= \int \frac{\lambda(3x+2)+\mu}{\sqrt{3x+2}} dx \\
&= \lambda \int \frac{3x+2}{\sqrt{3x+2}} dx + \mu \int \frac{1}{\sqrt{3x+2}} dx \\
&= \lambda \int (3x+2)^{\frac{1}{2}} dx + \mu \int (3x+2)^{-\frac{1}{2}} dx \\
&= \lambda \times \frac{(3x+2)^{\frac{3}{2}}}{\frac{3}{2} \times 3} + \mu \frac{(3x+2)^{\frac{1}{2}}}{\frac{1}{2} \times 3} + c \\
&= \frac{2}{3} \times \frac{2}{9} \times (3x+2)^{\frac{3}{2}} - \frac{1}{3} \times \frac{2}{3} (3x+2)^{\frac{1}{2}} + c \\
&= \frac{4}{27} \times (3x+2)^{\frac{3}{2}} - \frac{2}{9} \times (3x+2)^{\frac{1}{2}} + c \\
&= \frac{2}{9} \times \sqrt{3x+2} \left[\frac{2}{3} \times (3x+2) - 1 \right] + c \\
&= \frac{2}{9} \sqrt{3x+2} \left[\frac{6x+4-3}{3} \right] + c \\
&= \frac{2}{27} \sqrt{3x+2} (6x+1) + c \\
\therefore I &= \frac{2}{27} (6x+1) \sqrt{3x+2} + c
\end{aligned}$$

8. According to the question, we have to solve,

$$\cos^2 x \frac{dy}{dx} + y = \tan x$$

Therefore, on dividing both sides by $\cos^2 x$, the given equation can be rewritten as,

$$\begin{aligned}
\frac{dy}{dx} + \frac{y}{\cos^2 x} &= \frac{\tan x}{\cos^2 x} \\
\Rightarrow \frac{dy}{dx} + y \cdot \sec^2 x &= \tan x \sec^2 x
\end{aligned}$$

which is the linear differential equation of the form

$$\frac{dy}{dx} + Py = Q$$

Here, $P = \sec^2 x$ and $Q = \tan x \sec^2 x$

$$\therefore \text{IF} = e^{\int P dx} = e^{\int \sec^2 x dx} = e^{\tan x} \left[\because \int \sec^2 x dx = \tan x + C \right]$$

The solution of linear differential equation is given by

$$y \times \text{IF} = \int (Q \times \text{IF}) dx + C$$

$$\therefore y \times e^{\tan x} = \int \tan x \sec^2 x \cdot e^{\tan x} dx + C \dots (i)$$

Therefore, on putting $\tan x = t \Rightarrow \sec^2 x dx = dt$ in Eq.(i),

we get

$$ye^{\tan x} = \int t e^t dt + C$$

$$\Rightarrow ye^{\tan x} = t \int e^t dt - \int \left[\frac{d}{dt}(t) \int e^t dt \right] dt + C \text{ [using integration by parts]}$$

$$\Rightarrow ye^{\tan x} = te^t - \int 1 \times e^t dt + C$$

$$\Rightarrow ye^{\tan x} = te^t - e^t + C$$

$$\therefore ye^{\tan x} = \tan x e^{\tan x} - e^{\tan x} + C [\because t = \tan x]$$

On dividing both sides by $e^{\tan x}$, we get

$$y = \tan x - 1 + Ce^{-\tan x}$$

which is the required solution.

OR

It is clear that if a circle touches y-axis at the origin must have its centre on x-axis, because x-axis being at right angles to y-axis is the normal or line of radius of the circle.

Therefore, the centre of the circle is $(r, 0)$ where r is the radius of the circle.

$$\therefore \text{Equation of the required circle is } (x - r)^2 + (y - 0)^2 = r^2$$

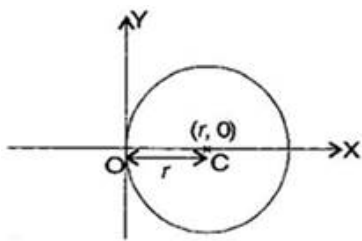
$$\Rightarrow x^2 + r^2 - 2rx + y^2 = r^2$$

$$\Rightarrow x^2 + y^2 = 2rx \dots (i)$$

Here r is the only arbitrary constant.

\therefore differentiating(i) w.r.t. x , we get

$$2x + 2y \frac{dy}{dx} = 2r \dots (ii)$$



Putting the value of $2r$ from eq. (ii) in eq. (i), we get

$$x^2 + y^2 = \left(2x + 2y \frac{dy}{dx}\right) x$$

$$\Rightarrow x^2 + y^2 = 2x^2 + 2xy \frac{dy}{dx}$$

$$\Rightarrow -2xy \frac{dy}{dx} - x^2 + y^2 = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} + x^2 - y^2 = 0$$

$$\Rightarrow 2xy \frac{dy}{dx} + x^2 = y^2, \text{ which is the required differential equation.}$$

9. To prove: $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$

Now we have, $|\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} + \vec{b})$ [$|\vec{x}|^2 = \vec{x} \cdot \vec{x}$]

$$\Rightarrow |\vec{a} + \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot \vec{a} + (\vec{a} + \vec{b}) \cdot \vec{b} \text{ [By distributivity of dot product over vector addition]}$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} + \vec{a} \cdot \vec{b} + \vec{b} \cdot \vec{b} \text{ [By distributivity of dot product over vector addition]}$$

$$\Rightarrow |\vec{a} + \vec{b}|^2 = |\vec{a}|^2 + 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 \dots\dots\dots(i) [\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}]$$

$$|\vec{a} - \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot (\vec{a} - \vec{b})$$
 [$|\vec{x}|^2 = \vec{x} \cdot \vec{x}$]

$$\Rightarrow |\vec{a} - \vec{b}|^2 = (\vec{a} + \vec{b}) \cdot \vec{a} + (\vec{a} + \vec{b}) \cdot (-\vec{b}) \text{ [By distributivity of dot product over vector addition]}$$

$$\Rightarrow |\vec{a} - \vec{b}|^2 = \vec{a} \cdot \vec{a} + \vec{b} \cdot \vec{a} - \vec{a} \cdot \vec{b} - \vec{b} \cdot \vec{b} \text{ [By distributivity of dot product over vector addition]}$$

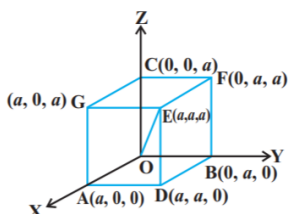
$$\Rightarrow |\vec{a} - \vec{b}|^2 = |\vec{a}|^2 - 2\vec{a} \cdot \vec{b} + |\vec{b}|^2 \dots\dots\dots(ii) [\because \vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}]$$

Adding (i) and (ii), we get

$$|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2(|\vec{a}|^2 + |\vec{b}|^2)$$

10. A cube is a rectangular parallelepiped having equal length, breadth and height.

Let OADBFEGC be the cube with each side of length a units.



The four diagonals are OE, AF, BG and CD.

The direction cosines of the diagonal OE which is the line joining two points O and E are

$$\frac{a-0}{\sqrt{a^2+a^2+a^2}}, \frac{a-0}{\sqrt{a^2+a^2+a^2}}, \frac{a-0}{\sqrt{a^2+a^2+a^2}}$$

$$\text{i.e., } \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$$

Similarly, the direction cosines of AF, BG and CD are $\frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$; $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ and $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}$ respectively.

Let l, m, n be the direction cosines of the given line which makes angles $\alpha, \beta, \gamma, \delta$ with OE, AF, BG, CD, respectively. Then

$$\cos \alpha = \frac{1}{\sqrt{3}}(l + m + n)$$

$$\cos \beta = \frac{1}{\sqrt{3}}(-l + m + n)$$

$$\cos \gamma = \frac{1}{\sqrt{3}}(l - m + n)$$

$$\cos \delta = \frac{1}{\sqrt{3}}(l + m - n)$$

Squaring and adding, we get

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma + \cos^2 \delta$$

$$= \frac{1}{3} [(l+m+n)^2 + (-l+m+n)^2] + (l-m+n)^2 + (l+m-n)^2$$

$$= \frac{1}{3} [4(l^2 + m^2 + n^2)] = \frac{4}{3} \text{ (as } l^2 + m^2 + n^2 = 1)$$

OR

Given equations of lines are,

$$\vec{r} = (\hat{i} + \hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k}) \dots \dots \dots \text{(i)}$$

$$\text{and } \vec{r} = (2\hat{i} + \hat{j} - \hat{k}) + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \dots \dots \dots \text{(ii)}$$

On comparing above equations with vector

equation $\vec{r} = \vec{a} + \lambda\vec{b}$, we get

$$\vec{a}_1 = \hat{i} + \hat{j} = 2\hat{i} - \hat{j} + \hat{k}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k}$$

$$\text{and } \vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k}$$

We know that, the shortest distance between two lines is given by

$$d = \frac{\left| \left(\vec{b}_1 \times \vec{b}_2 \right) \cdot \left(\vec{a}_2 - \vec{a}_1 \right) \right|}{\left| \vec{b}_1 \times \vec{b}_2 \right|} \dots \dots \dots \text{(iii)}$$

$$\text{Now, } \vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix}$$

$$= \hat{i}(-2+5) - \hat{j}(4-3) + \hat{k}(-10+3) \dots \dots \dots \text{(iv)}$$

$$\text{and } \left| \vec{b}_1 \times \vec{b}_2 \right| = \sqrt{(3)^2 + (-1)^2 + (-7)^2}$$

$$= \sqrt{9+1+49} = \sqrt{59} \dots \dots \dots \text{(v)}$$

$$\text{Also, } \vec{a}_2 - \vec{a}_1 = (2\hat{i} + \hat{j} - \hat{k}) - (\hat{i} + \hat{j})$$

$$= \hat{i} - \hat{k} \dots \dots \dots \text{(vi)}$$

From Eqs. (iii), (iv), (v) and (vi), we get

$$d = \left| \frac{(3\hat{i} - \hat{j} - 7\hat{k}) \cdot (\hat{i} - \hat{k})}{\sqrt{59}} \right|$$

$$\Rightarrow d = \left| \frac{3-0+7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}}$$

Hence, required shortest distance is $\frac{10}{\sqrt{59}}$ units.

Section C

11. Let $I = \int \frac{dx}{\sin x(3+2 \cos x)}$

Put $t = \cos x$

$$dt = -\sin x \, dx$$

$$\frac{dt}{-\sin x} = dx$$

$$\frac{dt}{-\sin x} = dx$$

$$= - \int \frac{dt}{\sin^2 x(3+2t)} = - \int \frac{dt}{(1-\cos^2 x)(3+2t)}$$

$$= - \int \frac{dt}{(1-t^2)(3+2t)}$$

$$\frac{1}{(1-t^2)(3+2t)} = \frac{1}{(1-t)(1+t)(3+2t)}$$

Now using partial fractions Putting $\frac{1}{(1-t)(1+t)(3+2t)} = \frac{A}{1-t} + \frac{B}{1+t} + \frac{C}{3+2t} \dots (1)$

$$A(1+t)(3+2t) + B(1-t)(3+2t) + C(1+t)(1-t) = 1$$

Now Putting $1+t=0$

$$t = -1$$

$$A(0) + B(2)(3-2) + C(0) = 1$$

$$B = \frac{1}{2}$$

Now Putting $1-t=0$

$$t = 1$$

$$A(2)(5) + B(0) + C(0) = 1$$



$$A = \frac{1}{10}$$

Now Putting $3 + 2t = 0$

$$t = -\frac{3}{2}$$

$$A(0) + B(0) + C\left(1 - \frac{9}{4}\right) = 1$$

$$C = \frac{-4}{5}$$

$$\frac{1}{(1-t)(1+t)(3+2t)} = \frac{1}{10} \times \frac{1}{1-t} + \frac{1}{2} \times \frac{1}{1+t} - \frac{4}{5} \times \frac{1}{3+2t}$$

$$\int \frac{1}{(1-t)(1+t)(3+2t)} dt = \frac{1}{10} \int \frac{1}{1-t} dt + \frac{1}{2} \int \frac{1}{1+t} dt - \frac{4}{5} \int \frac{1}{3+2t} dt$$

$$= -\frac{1}{10} \log |1-t| + \frac{1}{2} \log |1+t| - \frac{4}{5} \times \frac{\log |3+2t|}{2} + c$$

$$= -\frac{1}{10} \log |1 - \cos x| + \frac{1}{2} \log |1 + \cos x| - \frac{2}{5} \log |3 + 2 \cos x| + c$$

12. To find area $\{(x, y) : y^2 \leq 5x, 5x^2 + 5y^2 \leq 36\}$

The curves included are,

$$\Rightarrow y^2 = 5x \dots (i)$$

$$5x^2 + 5y^2 = 36$$

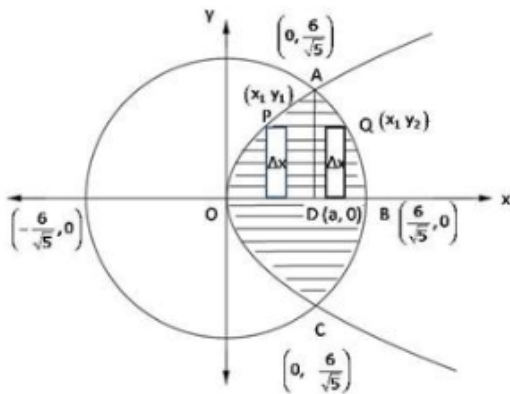
$$x^2 + y^2 = \frac{36}{5} \dots (ii)$$

Equation (i) represents a parabola with vertex (0, 0) and axis as x-axis.

Equation (ii) represents a circle with centre (0, 0) and radius $\frac{6}{\sqrt{5}}$ and meets axes at

$\left(\pm \frac{6}{\sqrt{5}}, 0\right)$ and $\left(0, \pm \frac{6}{\sqrt{5}}\right)$, x coordinate of point of intersection of circle and parabola is

a where, $a = \frac{-25 + \sqrt{1345}}{10}$, A rough sketch of curves is:-



Therefore, we have,

Required area = Region OCBAO

$$A = 2(\text{Region } \text{OBAO})$$

$$= 2(\text{Region } \text{ODAO} + \text{Region } \text{DBAD})$$

$$= 2 \left[\int_0^2 \sqrt{5x} dx + \int_2^{\frac{6}{\sqrt{5}}} \sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 - x^2} dx \right]$$

$$= 2 \left[\left(\sqrt{5} \frac{2}{3} x \sqrt{x} \right)_0^2 + \left(\frac{x}{2} \sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 - x^2} + \frac{36}{10} \sin^{-1} \left(\frac{x \sqrt{5}}{6} \right) \right)_2^{\frac{6}{\sqrt{5}}} \right]$$

$$= \frac{4\sqrt{5}}{3} a \sqrt{a} + 2 \left\{ \left(0 + \frac{18}{5} \cdot \frac{\pi}{2} \right) - \left(\frac{a}{2} \sqrt{\left(\frac{6}{\sqrt{5}}\right)^2 - a^2} + \frac{18}{5} \sin^{-1} \left(\frac{a \sqrt{5}}{6} \right) \right) \right\}$$

$$\text{Thus, } A = \frac{4\sqrt{5}}{3} a^{\frac{3}{2}} + \frac{18\pi}{5} - a \sqrt{\frac{36}{5} - a^2} - \frac{36}{5} \sin^{-1} \left(\frac{a \sqrt{5}}{6} \right)$$

$$\text{Where, } a = \frac{-25 + \sqrt{1345}}{10}$$

OR

Given circles are $x^2 + y^2 = 4 \dots (i)$

$$(x - 2)^2 + y^2 = 4 \dots (ii)$$

Eq. (i) is a circle with centre origin and

Radius = 2.

Eq. (ii) is a circle with centre C (2, 0) and



Radius = 2.

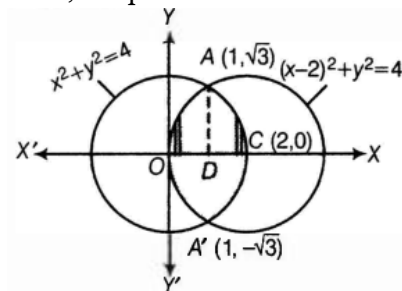
On solving Eqs. (i) and (ii), we get

$$\begin{aligned}(x-2)^2 + y^2 &= x^2 + y^2 \\ \Rightarrow x^2 - 4x + 4 + y^2 &= x^2 + y^2 \\ \Rightarrow x &= 1\end{aligned}$$

On putting $x = 1$ in Eq. (i), we get

$$y = \pm\sqrt{3}$$

Thus, the points of intersection of the given circles are $A(1, \sqrt{3})$ and $A'(1, -\sqrt{3})$.



Clearly, required area = Area of the enclosed region OACA'O between circles

$$= 2 [\text{Area of the region ODCAO}]$$

$$= 2 [\text{Area of the region ODAO} + \text{Area of the region DCAD}]$$

$$= 2 \left[\int_0^1 y_2 dx + \int_1^2 y_1 dx \right]$$

$$= 2 \left[\int_0^1 \sqrt{4 - (x-2)^2} dx + \int_1^2 \sqrt{4 - x^2} dx \right]$$

$$= 2 \left[\frac{1}{2}(x-2)\sqrt{4 - (x-2)^2} + \frac{1}{2} \times 4 \sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^1 + 2 \left[\frac{1}{2}x\sqrt{4 - x^2} + \frac{1}{2} \times 4 \sin^{-1} \frac{x}{2} \right]_1^2$$

$$= \left[(x-2)\sqrt{4 - (x-2)^2} + 4 \sin^{-1} \left(\frac{x-2}{2} \right) \right]_0^1 + \left[x\sqrt{4 - x^2} + 4 \sin^{-1} \frac{x}{2} \right]_1^2$$

$$= \left[\left\{ -\sqrt{3} + 4 \sin^{-1} \left(\frac{-1}{2} \right) \right\} - 0 - 4 \sin^{-1}(-1) \right] + \left[0 + 4 \sin^{-1} 1 - \sqrt{3} - 4 \sin^{-1} \frac{1}{2} \right]$$

$$= \left[(-\sqrt{3} - 4 \times \frac{\pi}{6}) + 4 \times \frac{\pi}{2} \right] + \left[4 \times \frac{\pi}{2} - \sqrt{3} - 4 \times \frac{\pi}{6} \right]$$

$$= (-\sqrt{3} - \frac{2\pi}{3} + 2\pi) + (2\pi - \sqrt{3} - \frac{2\pi}{3})$$

$$= \frac{8\pi}{3} - 2\sqrt{3} \text{ sq units.}$$

13. Given

$$\vec{r} = (\hat{i} + 2\hat{j} - 4\hat{k}) + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

$$\vec{r} = (3\hat{i} + 3\hat{j} - 5\hat{k}) + \mu(-2\hat{i} + 3\hat{j} + 8\hat{k})$$

Here, we have

$$\vec{a}_1 = \hat{i} + 2\hat{j} - 4\hat{k}$$

$$\vec{b}_1 = 2\hat{i} + 3\hat{j} + 6\hat{k}$$

$$\vec{a}_2 = 3\hat{i} + 3\hat{j} - 5\hat{k}$$

$$\vec{b}_2 = -2\hat{i} + 3\hat{j} + 8\hat{k}$$

Thus,

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 6 \\ -2 & 3 & 8 \end{vmatrix}$$

$$= \hat{i}(24 - 18) - \hat{j}(16 + 12) + \hat{k}(6 - 6)$$

$$\vec{b}_1 \times \vec{b}_2 = 6\hat{i} - 28\hat{j} + 0\hat{k}$$

$$\therefore |\vec{b}_1 \times \vec{b}_2| = \sqrt{6^2 + (-28)^2 + 0^2}$$

$$= \sqrt{36 + 784 + 9}$$

$$= \sqrt{829}$$

$$\vec{a}_2 - \vec{a}_1 = (3 - 1)\hat{i} + (3 - 2)\hat{j} + (-5 + 4)\hat{k}$$

$$\therefore \vec{a}_2 - \vec{a}_1 = 2\hat{i} + \hat{j} - \hat{k}$$

Now, we have

$$\begin{aligned}(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1}) &= (6\hat{i} - 28\hat{j} + 0\hat{k}) \cdot (2\hat{i} + \hat{j} - \hat{k}) \\&= (6 \times 2) + ((-28) \times 1) + (0 \times (-1)) \\&= 12 - 28 + 0 \\&= -16\end{aligned}$$

Thus, the shortest distance between the given lines is

$$\begin{aligned}d &= \left| \frac{(\vec{b_1} \times \vec{b_2}) \cdot (\vec{a_2} - \vec{a_1})}{|\vec{b_1} \times \vec{b_2}|} \right| \\ \Rightarrow d &= \left| \frac{-16}{\sqrt{820}} \right| \\ \therefore d &= \frac{16}{\sqrt{820}} \text{ units}\end{aligned}$$

CASE-BASED/DATA-BASED

14. Let A be the event of committing an error and E_1 , E_2 and E_3 be the events that Govind, Priyanka and Tahseen processed the form.

i. Using Bayes' theorem, we have

$$\begin{aligned}P(E_1 | A) &= \frac{P(E_1) \cdot P(A|E_1)}{P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + P(E_3) \cdot P(A|E_3)} \\&= \frac{0.5 \times 0.06}{0.5 \times 0.06 + 0.2 \times 0.04 + 0.3 \times 0.03} = \frac{30}{47} \\ \therefore \text{Required probability} &= P(\bar{E}_1 | A) \\&= 1 - P(E_1 | A) = 1 - \frac{30}{47} = \frac{17}{47}\end{aligned}$$

$$\begin{aligned}\text{ii. } \sum_{i=1}^3 P(E_i | A) &= P(E_1 | A) + P(E_2 | A) + P(E_3 | A) \\&= 1 \quad [\because \text{Sum of posterior probabilities is 1}]\end{aligned}$$